Lab: AP Review Sheets | AP Physics

Chapter 6: Gauss's Law | by Kaylin Yagura

Background: Gauss's Law relates to the distribution of electric charge to the resulting electric field. The law is expressed as $\oint E \cdot dA = Q/\varepsilon_0$, and it is This unit covers Coulomb's Law, Gauss's Law, Electric Field, Gaussian Surfaces, and Electric Flux.

Key Points and Phrases

1. Gaussian Surface is a hypothetical closed surface where Gauss's Law is applied
2. Gauss's Law only takes into account the total charge <u>enclosed</u> in the Gaussian Surface. This means that charge outside of the Gaussian Surface should not be factored into Q
3. It is important to look for symmetry in Gauss's Law problems because it can simplify calculations
4. Gauss's Law and Coulomb's Law are consistent with each other and can be used together.
5. Flux through a surface refers to how much of a vector field (in this case electrical) passes through the Gaussian surface

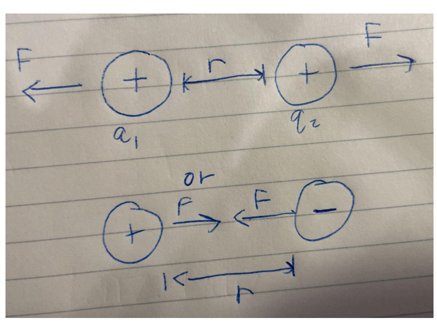
Diagrams and Pictures

Coulombs Law:

$$F = k\frac{q_1 q_2}{r^2}$$

- F is the magnitude of the force between two charges
- k is a constant
- $q1$ and $q2$ are the magnitudes of the two charges
- r is the radius

Used to find the electrostatic force between two charged particles.

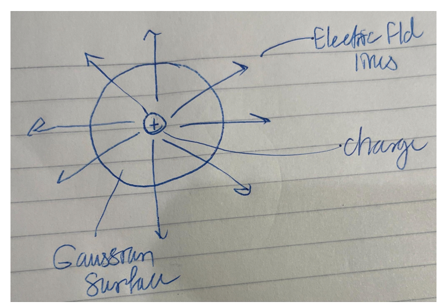


Formulas Necessary

Gauss's Law:
$\oint E \cdot dA = Q/\varepsilon_0$

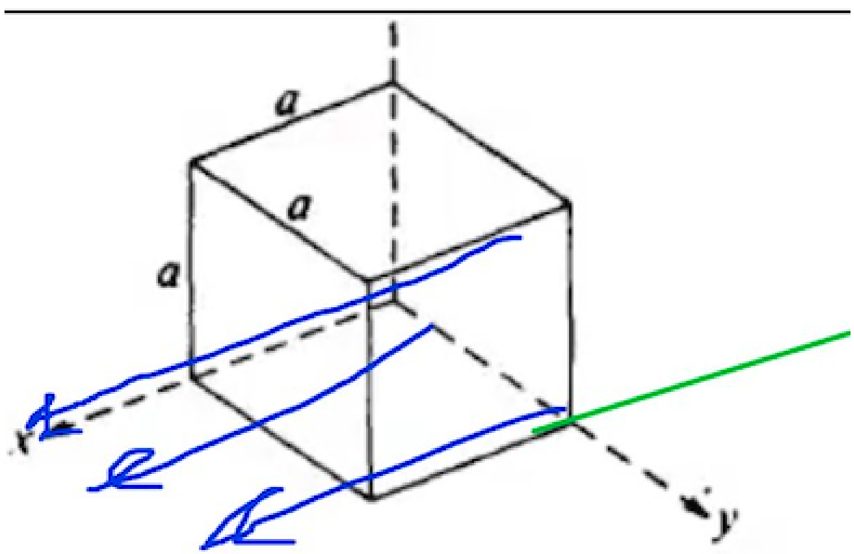
- $\oint E \cdot dA$ is the electric flux through a closed surface
- Q is the total charge enclosed by that surface
- ε_0 is a constant
- E is the electric field.

Useful for finding the electric field or the enclosed charge.



Practice Problems

1. [Easy] A closed surface, in the shape of a cube of side a, is oriented as shown above in a region where there is a constant electric field of magnitude E parallel to the x-axis. What is the total electric flux through the cubical surface?



(The blue lines depicted represent the electric field)

Solution:

There will be a total of zero electrical flux going through the cubical surface. Electric flux within any closed surface in region of uniform electric fields is zero because the total number of electric lines of the forces entering the closed surface equals that exiting the surface. Therefore, the total flux must be equal to zero.

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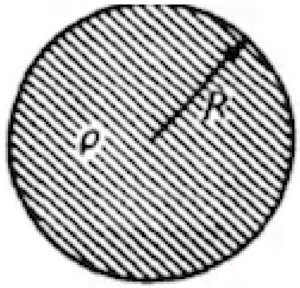
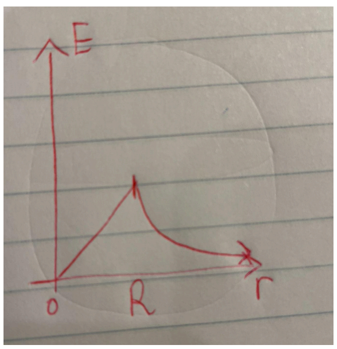
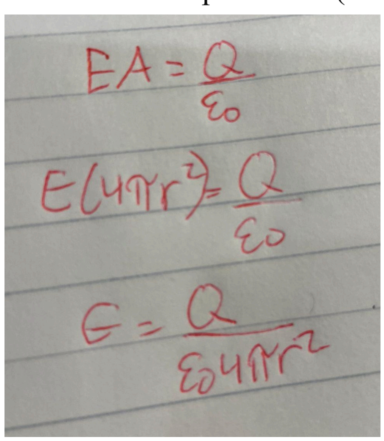
2. [Medium] The figure below shows a spherical distribution of charge of radius R and constant charge density ρ. Draw a graph that best represents the electric field strength E as a function of the distance r from the center of the sphere.



Answer:



The graph needs to start at 0, because there is no electric field strength when the distance from the center of the sphere is 0 (there isn't any enclosed charge).



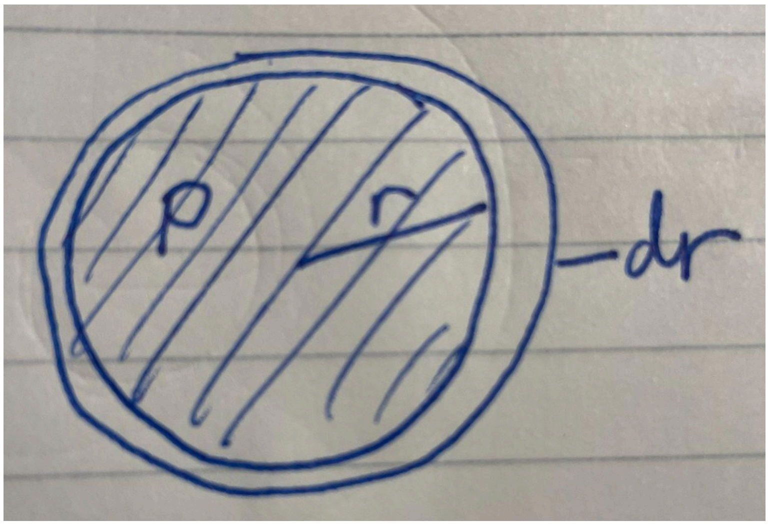
$$EA = \frac{Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{\varepsilon_0 4\pi r^2}$$

Because ρ is constant, as the radius increases, E constantly increases as well. This is because the enclosed charge Q steadily increases and according to the equation above measuring the electric field of the sphere, the electric field increases. Once the Gaussian surface surpasses the sphere however, there is no additional enclosed charge. This means that the radius as the radius increases, the eclectic field value becomes smaller (in a half parabolic shape because radius is squared) heading towards zero but not quite reaching it.

3. [Harder] A positive charge distribution exists within a nonconducting spherical region of radius a. The volume charge density ρ is not uniform but varies with the distance r from the center of the spherical charge distribution, according to the relationship ρ=**β**r for 0≤r≤a, where **β** is a positive constant, and ρ=0, for r>a. Show that the total charge Q in the spherical region of radius a is $\boldsymbol{\beta}\pi a^4$.



Solution:

$$Q = \int \rho \, dv$$
$$= \int_0^a \rho \, 4\pi r^2 dr$$
$$= \int_0^a 4\pi \beta r^3 dr$$
$$= 4\pi\beta \frac{r^4}{4} \Big|_0^a$$
$$= \beta\pi a^4$$

Using the knowledge that Q can be equal to the integral of ρdV, we can solve this equation. Since dV is equal to the $4\pi r^2$dr, we can substitute that into the equation and replace ρ with **β**r. Solving the integral shows that Q = $\boldsymbol{\beta}\pi a^4$.